University of California, Berkeley
Physics 110A Fall 2001 Section 1 (*Strovink*)

MIDTERM EXAMINATION 2

Directions: Do both problems, which have equal weight. This is a closed-book closed-note exam except for two $8\frac{1}{2} \times 11$ inch sheets containing any information you wish on both sides. A photocopy of the four inside covers of Griffiths is included with the exam. Calculators are not needed, but you may use one if you wish. Laptops and palmtops should be turned off. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Express your answer in terms of the quantities specified in the problem. Box or circle your answer.

Problem 1. (50 points)
A cylindrically symmetric region is bounded by $-\infty < z < \infty$ and $s < s_0$ (s is the cylindrical radius in Griffiths' notation). Within this region, the magnetic field may be obtained from the vector potential

$$\mathbf{A}(s) = \hat{z}\mu_0 C s^2 ,$$

where C is uniform, *i.e.* independent of $\mathbf{r}$. (You don't need to choose a particular gauge in order to work this problem, but, if it is helpful, you may work in Lorentz gauge $\nabla \cdot \mathbf{A} + \epsilon_0\mu_0 \partial V/\partial t = 0$.)

(**a**) (15 points)
For this part, take C to be a (positive) constant, *i.e.* independent of time t as well as $\mathbf{r}$. Calculate the current density $\mathbf{J}$, flowing within this region, that produces $\mathbf{A}$. The *direction* and *sign* of your answer are important. (In this application, note that

$$\tfrac{4\pi}{\mu_0}\mathbf{A}(\mathbf{r}) \neq \int \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\tau' ,$$

because the current-carrying region is infinite in extent.)

(**b**) (20 points)
For this part, take C to be a decaying function of time, *i.e.*

$$C(t) = C_0 \exp(-t/\tau) ,$$

where C_0 and τ are positive constants. Consider a rectangular loop drawn at constant azimuth ϕ, bounded by $0 < z < z_0$ and $0 < s < s_0$. Calculate the EMF $\mathcal{E}$ around this loop (the sign of your answer won't be graded).

(**c**) (15 points)
If you were asked to calculate the current density $\mathbf{J}$ for the conditions of part (**b**), where $\mathbf{A}$ decays with time, would you expect $\mathbf{J}$ to have the same dependence on s within our cylindrical region that you obtained in part (**a**)? Why or why not?

Problem 2. (50 points)
A nickel (five-cent coin) of radius a and thickness $d \ll a$ carries a uniform permanent magnetization

$$\mathbf{M} = \hat{z}M_0 ,$$

where M_0 is a positive constant and $\hat{z}$ is the nickel's axis of cylindrical symmetry.

(**a**) (30 points)
Calculate the magnetic field $\mathbf{B}(0,0,0)$ at the center of the nickel. The *direction* of $\mathbf{B}$ is important; express $\mathbf{B}$ to lowest nonvanishing order in d/a.

(**b**) (20 points)
In the plane $z = 0$, draw counterclockwise a large circular loop $s = b \gg a$ that is centered on the nickel. What magnetic flux Φ flows through this loop? The *sign* of Φ is important; express Φ to lowest nonvanishing order in d/b.